



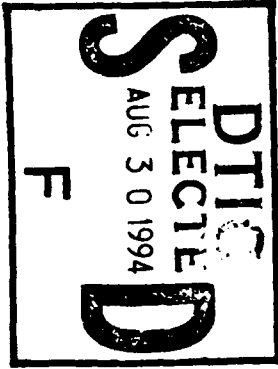
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System Identification Using Frequency Scanning and the Eigensystem Realization Algorithm

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The application of the eigensystem realization algorithm to flexible structures excited with nonimpulse type inputs is described. Sinusoidal pulses are a special subset of such inputs, and a discussion is given on their use to lock on to particular modes. The determination of impulse response functions from nonimpulse response functions is performed via a two step procedure. In the first step, free decay data is analyzed by the eigensystem realization algorithm to determine the structural frequencies and damping. Forced response data is analyzed in the second step, by solving an overdetermined set of linear equations for the modal coefficients of the impulse responses. It is shown that the number of impulse response coefficients is halved if displacement, velocity, and/or acceleration sensors are used in conjunction with force actuators. System identification is completed after the eigensystem realization algorithm is used on the impulse responses to produce a linear state space model of the structure. Numerical examples are given in which structural models are determined from simulated data corrupted with noise.

Introduction

THE eigensystem realization algorithm (ERA) is a system identification method that has been studied extensively.^{1,2} It determines a linear differential equation model of a structure by analyzing the structural impulse response data. The application of an impulse to a structure can be a problem, since it excites all of the structural modes. A model that is identified from this data would be larger than necessary, if the structure were to be operated in a band-limited manner.

In practice, one might apply a short pulse to the structure that would approximate an impulse for the low-frequency modes and would not excite the high-frequency ones. This procedure would be successful if the magnitude of the pulse were enough to produce a significant signal at the sensor outputs. If this is not the case, then a more frequency band-limited signal would be a logical choice of input. For example, several cycles of a sinusoidal wave at or near one of the resonant frequencies could be applied. A lightly damped structure may not survive extended excitation at one of the resonant frequencies; the sinusoidal pulse should be stopped after sufficient excitation has been achieved. By conducting a set of experiments involving sinusoidal pulses of different frequencies, the structural modes within a certain frequency range can be observed. Finite element analysis and previous system identifications can be used to narrow down the frequency bands of interest. ERA cannot be used to analyze this nonimpulse response data; an algorithm is needed to determine the impulse response from nonimpulse response data. One such algorithm was developed in recent work by Horta et al.,³ Phan et al.,⁴ Juang et al.,⁵ and Phan et al.⁶ The algorithm, known

as Observer/Kalman Filter Identification, identifies an observer/Kalman filter to determine the impulse response.

The remainder of this paper describes an alternate method that does not involve an observer/Kalman filter. Instead, ERA is first used to determine the frequencies and damping from the unforced response. The impulse response can then be found from the least squares solution of an overdetermined set of linear equations. The impulse responses between each input and output pair are then simultaneously analyzed by ERA to produce a linear model of the structure.

Derivation of Procedure

Consider the following model of a flexible structure:

$$M\ddot{q} + C\dot{q} + Kq = \beta u$$

$$y_d = C_d q, \quad y_v = C_v \dot{q}, \quad y_a = C_a \ddot{q} \quad (1)$$

where the displacement q , the input force u , the displacement sensor output y_d , the velocity output y_v , and the acceleration output y_a are dimensioned as: $q \in R^n$, $u \in R^m$, $y_d \in R^p$, $y_v \in R^p$, $y_a \in R^p$.

Without loss of generality, we will consider each input-output pair separately ($m = 1$, $p = 1$). The single output is assumed to be a displacement, a velocity, or an acceleration. The mass matrix M is assumed to be positive definite and symmetric; the damping matrix C and the stiffness matrix K are assumed to

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be positive semidefinite and symmetric. It is assumed that the system is modalizable,

$$\ddot{v} + C'\dot{v} + K'v = \beta'u$$

$$y_d = C_d'v, \quad y_v = C_v'\dot{v}, \quad y_a = C_a'\ddot{v} \quad (2)$$

The matrices C' and K' are diagonal. For undamped systems and for systems with Rayleigh damping, modalization can be performed. This modalizability assumption is not essential for the system identification method described subsequently, but it will be seen that the number of parameters which need to be determined is reduced if modalization occurs.

Consider one of the modes and the contribution to the output from this mode,

$$\ddot{v}_i + c_i'\dot{v}_i + k_i'v_i = \beta_i'u$$

$$y_d = c_d'v_i, \quad y_v = \sum_{i=1}^n y_{vi}, \quad y_a = \sum_{i=1}^n y_{ai} \quad (3)$$

$$y_{vi} = c_v'\dot{v}_i, \quad y_{ai} = c_a'\ddot{v}_i$$

$$= -c_a'c_i'\dot{v}_i - c_a'k_i'v_i + c_a'\beta_i'u$$

$$y_a = \sum_{i=1}^n y_{ai}$$

Since it is possible to perform the transformation $v_i = \beta_i'u_i$ and then cancel β_i' from the given differential equation, we will assume from this point onward that this has been done by setting β_i' equal to 1.

The impulse response from this second-order system is given by

$$h_{ri}(t) = c_i e^{s_i t} + c_i^* e^{s_i^* t} + c_a' \delta(t) \quad (4)$$

$$s_i = \sigma_{ri} + j\omega_{ri}, \quad c_i = c_{i1} + jc_{i2}$$

where the asterisk is the complex conjugate operator; c_i and s_i are the complex modal coefficient and the continuous time eigenvalue, respectively; and $\delta(t)$ is the Dirac delta function. A first-order state space system that has this impulse response is given by

$$\dot{x}_i(t) = A_i x_i(t) + B_i u(t)$$

$$y_i(t) = C_i x_i(t) + D_i u(t) \quad (5)$$

$$A_i = \begin{pmatrix} s_i & 0 \\ 0 & s_i^* \end{pmatrix}, \quad B_i = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$C_i = (c_i \quad c_i^*), \quad D_i = c_a'$$

The system described by Eq. (3) can also be placed in a first-order state space form

$$\dot{x}_i(t) = A_i x_i(t) + B_i u(t)$$

$$y_i(t) = C_i x_i(t) + D_i u(t) \quad (6)$$

$$x_i(t) = \begin{pmatrix} v_i(t) \\ \dot{v}_i(t) \end{pmatrix}$$

$$A_i = \begin{pmatrix} 0 & 1 \\ -k_i' & -c_i' \end{pmatrix}, \quad B_i = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad D_i = c_a'$$

For a displacement sensor, $C_i = (c_d' \ 0)$. For a velocity sensor,

$C_i = (0 \ c_v')$. For an acceleration sensor, $C_i = (-c_a' k_i' - c_a' c_i')$. The system described by Eq. (5) is the diagonalized version of the system described by Eq. (6). Therefore, the transfer functions are equal,

$$C_i(sI - A_i)^{-1}B_i = C_i(sI - A_i)^{-1}B_i \quad (7)$$

After some algebra, the Eq. (7) yields conditions for the displacement and the velocity sensors.

Displacement sensor:

$$c_{i1} = 0 \quad (8)$$

Velocity sensor:

$$\omega_{ri} c_{i2} = -\sigma_{ri} c_{i1} \quad (9)$$

Acceleration sensor:

$$I_{i1} c_{i1} = I_{i2} c_{i2} \quad (10)$$

$$c_a' = 2c_{i2} \frac{\sigma_{ri} I_{i2} / I_{i1} + \omega_{ri}}{\sigma_{ri}^2 + \omega_{ri}^2} \quad (11)$$

where the constants c_{i1} , c_{i2} , σ_{ri} , and ω_{ri} were defined in Eq. (4), and the new constants I_{i1} and I_{i2} are defined as follows:

$$I_{i1} = 1 - [2\sigma_{ri}^2 / (\sigma_{ri}^2 + \omega_{ri}^2)] \quad (12)$$

$$I_{i2} = [2\sigma_{ri}\omega_{ri} / (\sigma_{ri}^2 + \omega_{ri}^2)] \quad (13)$$

Now consider the discrete time version of the system described by Eq. (5)

$$x_{ri}(k+1) = F_{ri} x_{ri}(k) + G_{ri} u(k)$$

$$y_i(k) = H_{ri} x_{ri}(k) + \Phi_i u(k) \quad (14)$$

$$F_{ri} = \begin{pmatrix} r_i & 0 \\ 0 & r_i^* \end{pmatrix}, \quad G_{ri} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$H_{ri} = (c_i \quad c_i^*), \quad \Phi_i = c_a'$$

$$r_i = e^{s_i h} \quad (15)$$

$$= \sigma_{ri} + j\omega_{ri} \quad (16)$$

$$= m_i e^{j\theta_i} \quad (17)$$

where h is the time step size. If an impulse is applied at $k = 0$ to this discrete time system, then the discrete time impulse response is equal to the impulse response of the continuous time system (5) at the sampling points:

$$h_{ri}(k) = c_i r_i^k + c_i^* (r_i^*)^k \quad (18)$$

$$= c_a' \quad (k = 0) \quad (19)$$

This is also the form of the unforced response (nonzero initial conditions and zero input). Therefore, an analysis of the unforced response by ERA will correctly determine the discrete time eigenvalues. The eigenvalues of the state space model produced by ERA are the discrete time eigenvalues r_i . By performing a complex logarithm on Eq. (15), we can obtain the continuous time eigenvalues from the discrete time ones.

After substituting for c_i [Eq. (4)] and r_i [Eq. (17)] into the equation (18) and simplifying, we obtain

$$h_{r_i}(k) = c_{i1}\gamma_{i1}(k) + c_{i2}\gamma_{i2}(k) \quad (20)$$

$$= c'_{ai} \quad (k = 0)$$

$$\gamma_{i1}(k) = 2m_i^{\dagger} \cos(k\theta_i) \quad (21)$$

$$\gamma_{i2}(k) = -2m_i^{\dagger} \sin(k\theta_i)$$

Using conditions (8), (9), and (10), we can simplify the equation (20) for $k > 0$ if the modal frequency ω_{ri} is not equal to zero, and if I_{ii} is not equal to zero.
Displacement sensor:

$$h_{r_i}(k) = c_{i2}[\gamma_{i2}(k)] \quad (22)$$

Velocity sensor:

$$h_{r_i}(k) = c_{i1}[\gamma_{i1}(k) - (\sigma_{ri}/\omega_{ri}) \gamma_{i2}(k)] \quad (23)$$

Acceleration sensor:

$$h_{r_i}(k) = c_{i2}[(I_{ii}/I_{ii}) \gamma_{i1}(k) + \gamma_{i2}(k)] \quad (24)$$

The displacement output is equal to the convolution of the impulse response (from all of the modes) with the input force, provided there are zero initial conditions

$$y_d(k) = h_r(k) * u(k) \cdot h \quad (25)$$

$$= \left[\sum_{i=1}^n h_{r_i}(k) \right] * u(k) \cdot h \quad (26)$$

$$= \sum_{i=1}^n c_{i2}[\gamma_{i2}(k) * u(k) \cdot h] \quad (27)$$

where $\cdot h$ indicates multiplication by the time step size.

A system identification experiment may run from $k = 0$ to $k = L$. If the number of data points (L) is larger than the number of modes (n) then Eq. (27) is an overdetermined set of linear equations which is to be solved for the coefficients c_{i2} ($i = 1, \dots, n$). The QR factorization can be used to accurately obtain a least squares solution to an overdetermined set of linear equations, as described in Ref. 7.

The response for a velocity sensor (for zero initial conditions) can be derived similarly and is given by

$$y_v(k) = \sum_{i=1}^n c_{i1} \left[\gamma_{i1}(k) - \frac{\sigma_{ri}}{\omega_{ri}} \gamma_{i2}(k) \right] * u(k) \cdot h \quad (28)$$

with c_{i1} ($i = 1, \dots, n$) being the coefficients to be solved for.

Likewise, the response for an acceleration sensor (for zero initial conditions) is given by

$$y_a(k) = \sum_{i=1}^n c_{i2} \left[\frac{I_{ii}}{I_{ii}} \gamma_{i1}(k) + \gamma_{i2}(k) \right] * u(k) \cdot h \quad (29)$$

After the c_{i1} or c_{i2} coefficients have been determined for each input-output pair, the impulse response for each input-output pair can be computed for $k = 0, \dots, L$ by using Eqs. (22-24) and summing over all of the modes. The eigensystem realization algorithm can then be used on all of the impulse responses simultaneously, to yield a linear state space model of the flexible structure (see Ref. 1 for details).

Real Matrix Representation

A state variable transformation can be used to produce a system representation of Eq. (14), which is composed of real matrices

$$x_{r_i}(k) = T x'_{r_i}(k) \quad (30)$$

$$T = \begin{pmatrix} j & 1 \\ -j & 1 \end{pmatrix} \quad (31)$$

$$F'_{r_i} = T^{-1} F_{r_i} T \quad (32)$$

$$= \begin{pmatrix} \sigma_{r_i} & \omega_{r_i} \\ -\omega_{r_i} & \sigma_{r_i} \end{pmatrix} \quad (33)$$

$$G'_{r_i} = T^{-1} G_{r_i} \quad (34)$$

$$= \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (35)$$

$$H'_{r_i} = H_{r_i} T \quad (36)$$

$$= (-2c_{i2} \quad 2c_{i1}) \quad (37)$$

Numerical Examples

Single Input-Single Output

The identification of two structural modes is simulated for a displacement sensor. From the discussion in the previous section and the displacement sensor condition (8), the following is an example of a state space representation for a single input-single output structure:

$$F_1 = \begin{pmatrix} 0.960 & 0.282 & 0 & 0 \\ -0.282 & 0.960 & 0 & 0 \\ 0 & 0 & 0.978 & 0.208 \\ 0 & 0 & -0.208 & 0.978 \end{pmatrix}$$

$$G_1 = \begin{pmatrix} 0 \\ 0.002 \\ 0 \\ 0.002 \end{pmatrix}$$

$$H_1 = (7 \quad 0 \quad 7 \quad 0) \quad \Phi_1 = 0$$

As the F_1 matrix shows, the discrete time eigenvalues are at $0.960 \pm j0.282$ and at $0.978 \pm j0.208$. Any function could have been applied to the input, but for the purpose of this example a sinusoidal pulse was applied (Fig. 1). The displace-

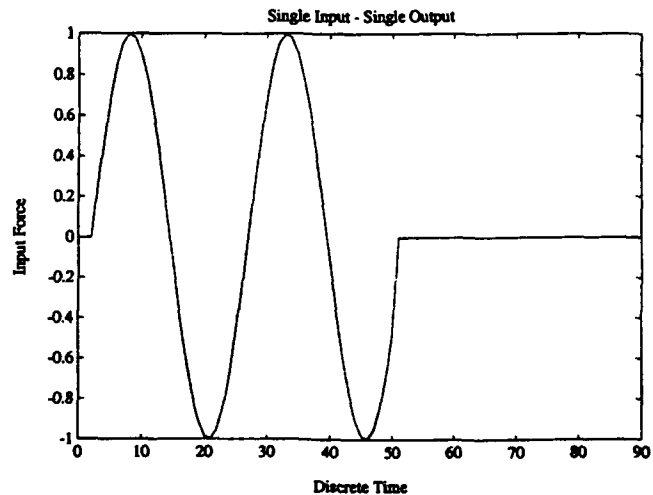


Fig. 1 Input excitation.

ment output with 5% added noise is shown in Fig. 2. Since the input is zero for $k > 50$, the eigensystem realization algorithm was applied to the unforced response in Fig. 2 for $k > 50$ to determine the modal damping and frequencies. The impulse response coefficients in Eq. (27) were solved for, and the resulting modalized state space system is given by

$$F_2 = \begin{pmatrix} 0.95 & 0.37 & 0 & 0 \\ -0.28 & 0.959 & 0 & 0 \\ 0 & 0 & 0.974 & 0.204 \\ 0 & 0 & -0.204 & 0.974 \end{pmatrix}$$

$$G_2 = \begin{pmatrix} 0 \\ 0.002 \\ 0 \\ 0.002 \end{pmatrix}$$

$$H_2 = (6.49 \ 0 \ 8.17 \ 0) \quad \Phi_2 = 0$$

The worst error occurs in the comparison between the third elements of the H_1 matrix and H_2 matrix; the error is 7%.

Two Inputs-Two Outputs

As another example, consider the case of two structural modes with identical frequencies. The following is a discrete time system for two displacement sensors and two force actuators:

$$F_1 = \begin{pmatrix} 0.951 & 0.309 & 0 & 0 \\ -0.309 & 0.951 & 0 & 0 \\ 0 & 0 & 0.951 & 0.309 \\ 0 & 0 & -0.309 & 0.951 \end{pmatrix}$$

$$G_1 = \begin{pmatrix} 0 & 0 \\ 2 \times 10^{-3} & 10^{-3} \\ 0 & 0 \\ 5 \times 10^{-4} & 2 \times 10^{-3} \end{pmatrix}$$

$$H_1 = \begin{pmatrix} 7 & 0 & 3 & 0 \\ 5 & 0 & 7 & 0 \end{pmatrix} \quad \Phi_1 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

As the F_1 matrix shows, the repeated discrete time eigenvalues are at $0.951 \pm j0.309$. In the first experiment, a sinusoidal pulse

is applied to the first input (Fig. 3), with the second input being zero. The displacement outputs with 10% added noise are shown in Fig. 4. In the second experiment, a sinusoidal pulse is applied to the second input (Fig. 3), with the first input being zero. The displacement outputs with 10% added noise are shown in Fig. 5.

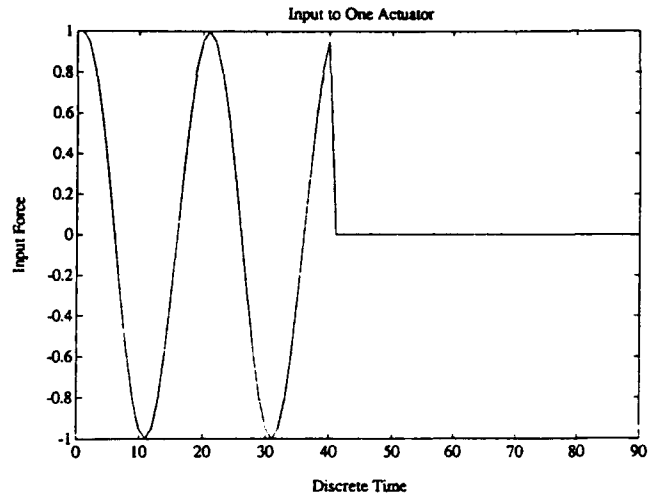


Fig. 3 Input excitation at one actuator.

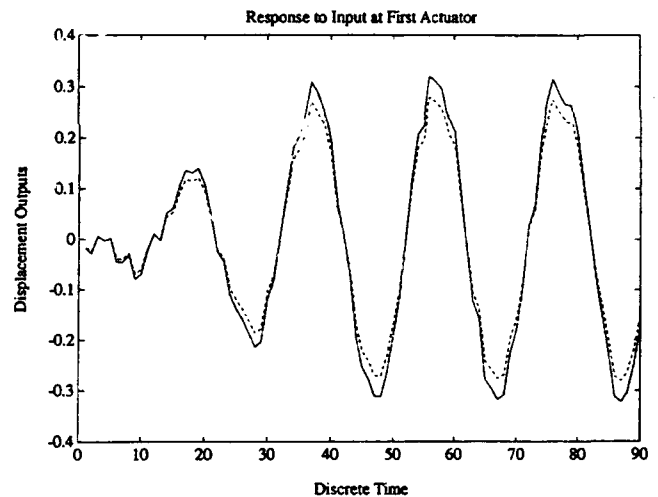


Fig. 4 Noisy outputs due to first actuator.

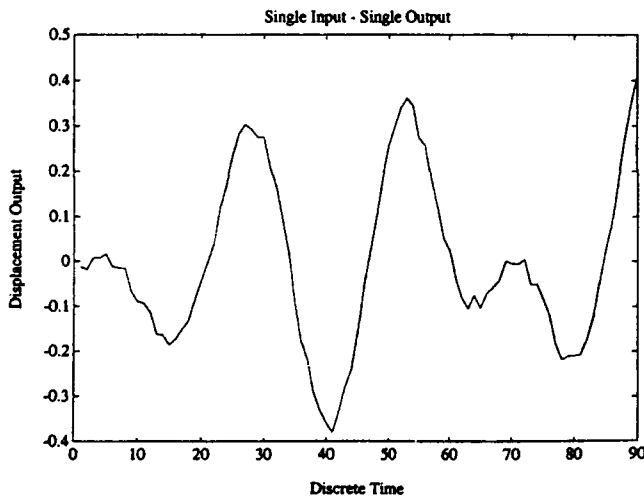


Fig. 2 Noisy output.

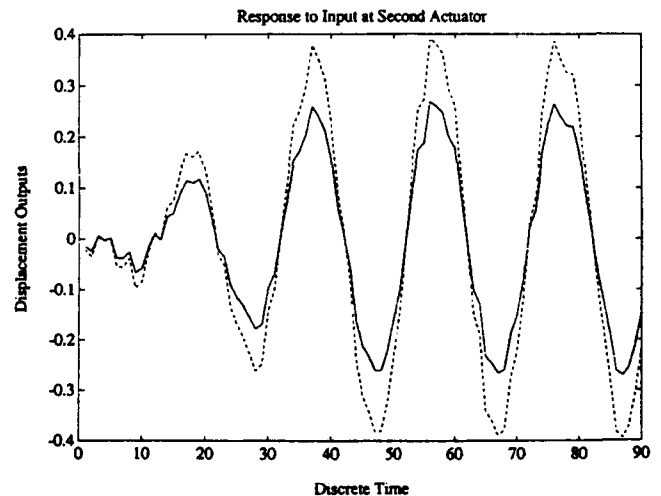


Fig. 5 Noisy outputs due to second actuator.

Since the input is zero for $k > 41$, the eigensystem realization algorithm was applied to the unforced response in Fig. 4 and 5 for $k > 41$ to determine the modal damping and frequencies. The impulse response coefficients in Eq. (27) were solved for. With these coefficients, the four impulse response functions between the two inputs and two outputs were computed using Eq. (22). The eigensystem realization algorithm was then used on the four impulse response functions to obtain a discrete time state space model. After a modalizing transformation of that state space model, we obtain

$$F_2 = \begin{pmatrix} 0.950 & 0.308 & 0 & 0 \\ -0.308 & 0.950 & 0 & 0 \\ 0 & 0 & 0.950 & 0.308 \\ 0 & 0 & -0.308 & 0.950 \end{pmatrix}$$

$$G_2 = \begin{pmatrix} 4.13 \times 10^{-11} & 4.75 \times 10^{-11} \\ 2.11 \times 10^{-3} & 1.05 \times 10^{-3} \\ -4.84 \times 10^{-11} & -5.40 \times 10^{-11} \\ 5.27 \times 10^{-4} & 2.11 \times 10^{-3} \end{pmatrix}$$

$$H_2 = \begin{pmatrix} 7.00 & -5.50 \times 10^{-8} & 3.00 & -5.35 \times 10^{-8} \\ 5.00 & 5.26 \times 10^{-8} & 7.00 & 4.03 \times 10^{-8} \end{pmatrix}$$

$$\Phi_2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

In comparing the parameters of the first model with the identified model, we find that the error is less than 6%.

Discussion

System identification using nonimpulsive inputs has been described in this paper. Band-limited white noise could be used as the nonimpulsive input, but the sinusoidal pulse was considered because of its simplicity. An initial frequency range for the sinusoidal pulse frequency can be determined via finite element analysis and previous system identifications. The frequency range is divided into a grid with the desired frequency resolution, and a set of system identifications is performed for each grid point. Any mode that is detected by one of these system identifications can be identified once more, by setting the frequency of the sinusoidal pulse to the identified mode frequency. This causes the sinusoidal pulse to "lock on" to the detected mode, thereby maximizing the signal-to-noise ratio for that particular mode. The system identified model increases by at least one mode whenever a lock-on condition occurs.

Passive vibration control is well suited for use with this form of modal system identification. Analytical formulas have been derived for passive vibration suppression by an inertial reaction device,⁸ or by piezoelectric actuators.⁹ After the sinusoidal pulse excitation, the modes at or near that frequency can be identified via the procedure in this paper. With this identified model, the analytical formulas can be used to determine a passive vibration controller. This ease of design and the unconditional stability of passive controllers provide a means to suppress the modes that are excited by a sinusoidal pulse at a particular frequency. Rapid suppression is desirable, so that the system identification can quickly proceed to the next frequency data point.

If modalizability of the second-order structural differential equations can be assumed, the number of impulse response coefficients to be determined is reduced by one-half. This removal of parameters ensures that certain physical constraints are satisfied. For example, a displacement sensor on a flexible structure with zero initial energy must show zero displacement at time zero, even when a force impulse is applied at time zero. The first modal coefficient c_1 is absent in the modal component of the displacement impulse response [Eq. (22)]; the presence of the sine function in the equation causes the modal displacement to be zero at time zero. Since the displacement impulse

response is the sum of the modal displacement impulse responses, it is guaranteed to be zero at time zero. The situation for velocity or acceleration sensors is somewhat similar, in that only one of the two modal coefficients is to be solved for.

In the case where the damping is large and modalizability does not occur, there is no relationship between the two modal coefficients for each mode. The simplifications presented in the given analysis are not performed in this case, and both modal coefficients are solved for. It is still possible, however, to constrain the displacement impulse response to be zero at time zero. Setting the discrete time k to zero in the expression for the displacement impulse response will result in a linear equation that constrains the modal coefficients. This equation is to be appended to the overdetermined set of linear equations described in this paper, and the whole set of equations is to be solved for the modal coefficients. In the case of a velocity sensor, the impulse response expression contains velocity modal coefficients. Integrating the expression results in a displacement impulse response, which is constrained to be zero at time zero. Thus a linear constraint is established between the velocity modal coefficients; this is appended to the overdetermined set of equations. A linear constraint is similarly obtained in the acceleration sensor case by performing two integrations on the impulse response expression.

Structural models were considered in this paper; the more general case of a set of first-order differential equations can be considered as well. In this case, terms of the form $c_i r_i^k$ appear in the discrete time impulse response, where r_i is the i th discrete time eigenvalue and c_i is a coefficient. However, terms of the form $c_i k^j r_i^k$ are necessary if Q repeated eigenvalues occur ($0 \leq j < Q$). The combination of the eigensystem realization algorithm and a least squares determination of the impulse response coefficients would also determine a linear state space model, for the case of first-order differential equations.

For multi-input/multi-output systems, the proposed procedure requires one input excitation at a time and that experiments be done separately for each input-output pair. Simultaneous use of all inputs can be advantageous in that only one experiment need be performed for all inputs and outputs. However, one experiment provides less data, which would tend to increase the sensitivity of the system identified model to noise. If it is critical to perform only one experiment, then the system identification procedure would need to be slightly modified. The impulse response expressions would be expanded to sum the modal components from each input, and more modal coefficients would be simultaneously solved for.

A more significant reason for the simultaneous use of all inputs is to increase the excitation of the structure. In the case of many weak actuators distributed over a large structure, the use of only one actuator may not produce significant sensor signals. An input transformation can be performed where new inputs are defined to be linear combinations of the physical inputs. Experiments would use only one of the new inputs at a time, but this would imply the simultaneous use of all of the physical inputs. With the significant sensor signals resulting from the simultaneous input usage, a model is identified that relates the new set of inputs to the sensor outputs. A model relating the physical inputs to the sensor outputs can be obtained by inverting the input transformation.

Concluding Remarks

A method has been presented that applies the eigensystem realization algorithm to flexible structures excited with nonimpulsive type inputs. The simplicity of the method makes it an interesting alternative to the OKID method. However, it is more complex than the determination of the impulse response via the inverse Fourier transform of the frequency response function.

Both steps of the presented method possess least squares characteristics that minimize the sensitivity of the identified model to noise. The inverse Fourier transform method, with its simplicity, does not attempt to minimize noise sensitivity. A

quantitative comparison of these three methods with respect to simplicity and noise sensitivity, therefore, seems appropriate for future research.

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